

Topic 5: Geometry and Trigonometry

Topic number	Content
5.1	Equation of a line in two dimensions: the forms $y = mx + c$ and $ax + by + d = 0$ Gradient; intercepts Points of intersection of lines Lines with gradients, m_1 and m_2 Parallel lines $m_1 = m_2$ Perpendicular lines, $m_1 \times m_2 = -1$
5.2	Use of sine, cosine and tangent ratios to find the sides and angles of right-angled triangles. Angles of elevation and depression
5.3	Use of the sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ Use of the cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$; $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ Use of area of a triangle $area = \frac{1}{2}ab \sin C$ Construction of labeled diagrams from verbal statements.
5.4	Geometry of three-dimensional solids: cuboid; right prism; right pyramid; right cone; cylinder; sphere; hemisphere; and combinations of these solids The distance between two points: for example: between two vertices or vertices with midpoints or midpoints with midpoints The size of an angle between two lines or between a line and a plane
5.5	Volume and surface areas of the three dimensional solids defined in 5.4

Brain Dump

Soh-Cah-Toa

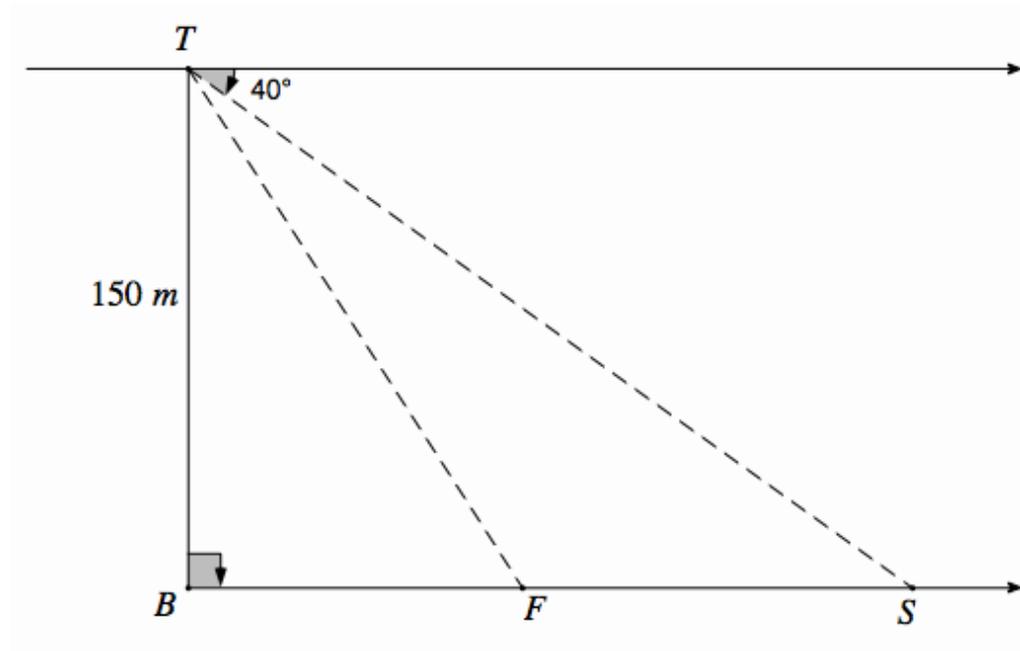
$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

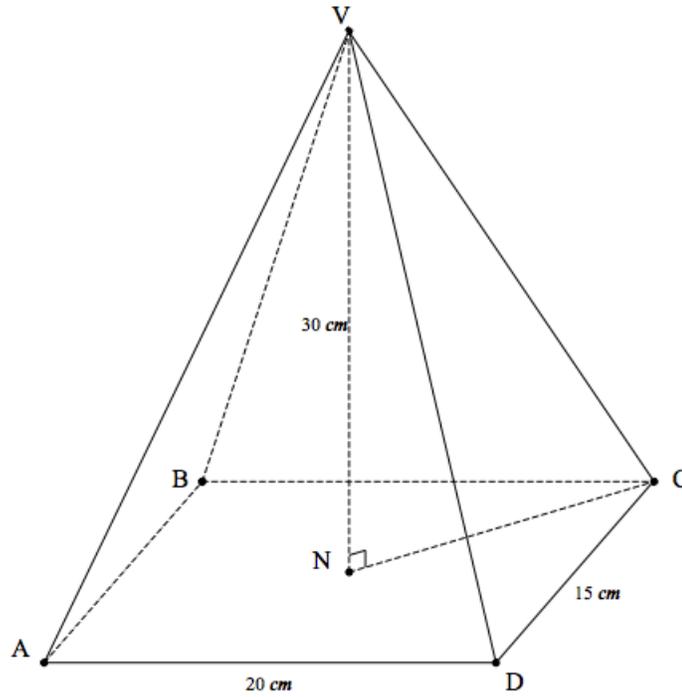
1. Tom stands at the top, T , of a vertical cliff 150 m high and sees a fishing boat, F , and a ship, S . B represents a point at the bottom of the cliff directly below T . The angle of depression of the ship is 40° and the angle of depression of the fishing boat is 55° .

Diagram not to scale



- Calculate, SB , the distance between the ship and the bottom of the cliff. (2 marks)
- Calculate, SF , the distance between the ship and the fishing boat, Give your answer correct to the nearest metre. (4 marks)

2. The diagram shows a rectangular based right pyramid $VABCD$ in which $AD = 20\text{ cm}$, $DC = 15\text{ cm}$ and the height of the pyramid, $VN = 30\text{ cm}$.



Calculate

a) i) the length of AC :

ii) The length of VC .

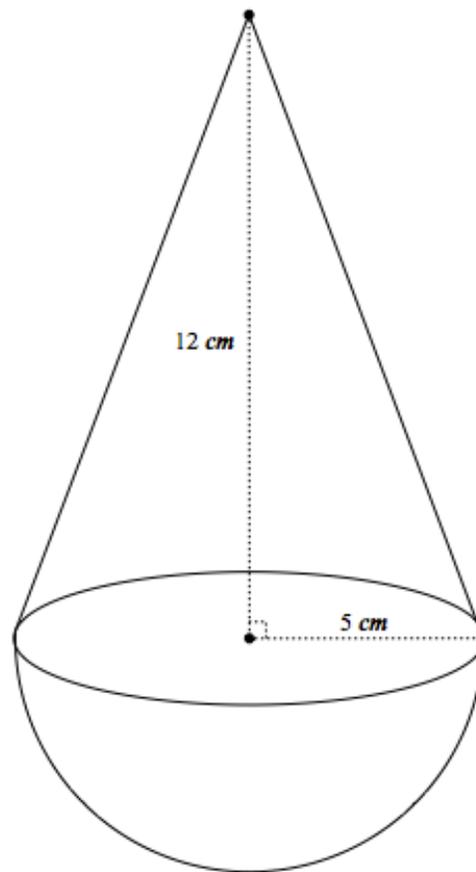
(4 marks)

b) The angle between VC and the base $ABCD$.

(2 marks)

3. A child's toy consists of a hemisphere with a right circular cone on top. The height of the cone is 12 cm and the radius of its base is 5 cm. The toy is painted red.

Diagram not to scale

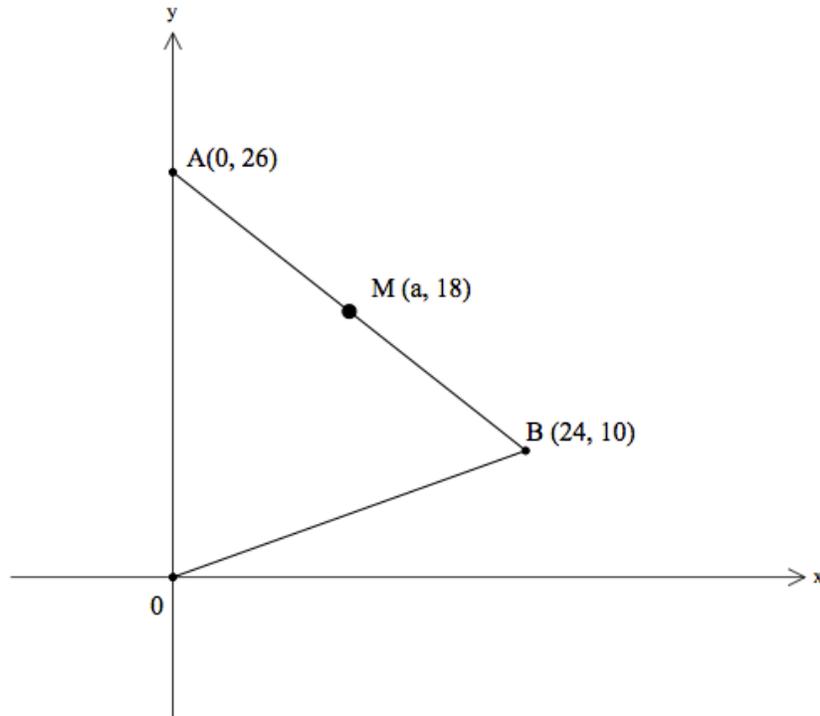


a) Calculate the length, l , of the slant height of the cone. (2 marks)

b) Calculate the area that is painted red. (4 marks)

4. The diagram shows the points $M(a, 18)$ and $B(24, 10)$. The straight line BM intersects the y -axis at $A(0, 26)$. M is the midpoint of the line segment AB .

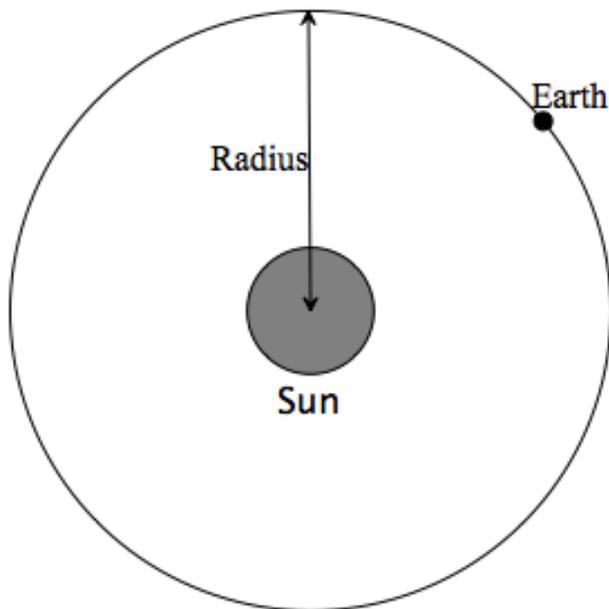
Diagram not to scale



- a) Write down the value of a . (1 marks)
- b) Find the gradient of the line AB . (2 marks)
- c) Decide whether triangle OAM is a right-angled triangle. Justify your answer. (3 marks)

5. The average radius of the orbit of the Earth around the Sun is 150 million kilometers.

Diagram not to scale



- a) Write down this radius, in kilometres, in the form $a \times 10^k$, where $1 \leq a < 10$, $k \in \mathbb{Z}$.
(2 marks)

The Earth travels around the Sun in one orbit. It takes one year for the Earth to complete one orbit.

- b) Calculate the distance, in kilometres, the Earth travels around the Sun in one orbit, assuming that the orbit is a circle.
(2 marks)

Today is Anna's 17th birthday.

- c) Calculate the total distance that Anna has travelled around the Sun, since she was born.
(2 marks)

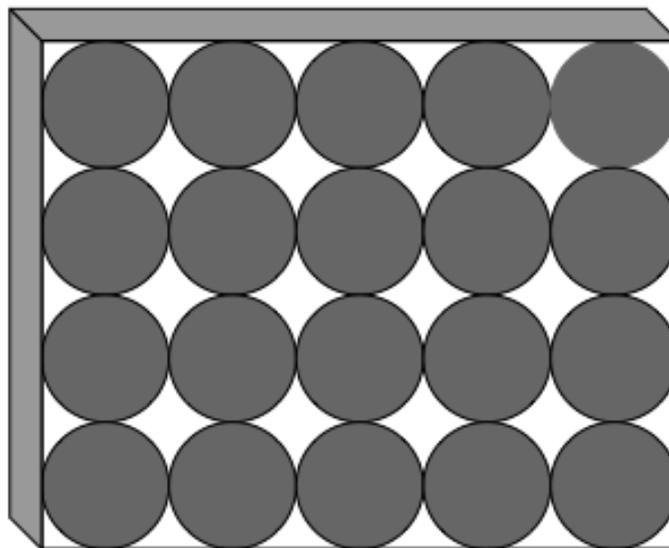
6. Chocolates in the shape of spheres are sold in boxes of 20.

Each chocolate has a radius of 1 cm.

a) Find the volume of 1 chocolate. (2 marks)

b) Write down the volume of 20 chocolates. (1 mark)

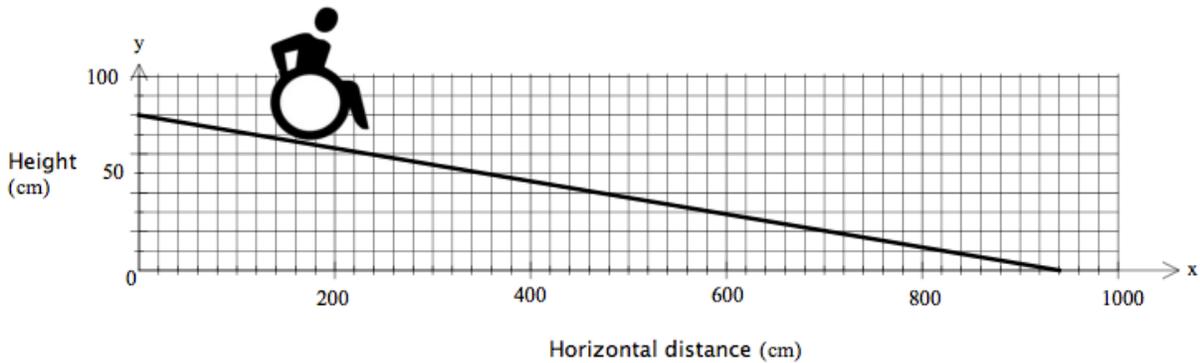
The diagram shows the chocolate box from above. The 20 chocolates fit perfectly in the box with each chocolate touching the ones around it or the sides of the box.



c) Calculate the volume of the box. (2 marks)

d) Calculate the volume of empty space in the box. (1 mark)

7. The diagram shows a wheelchair ramp, *A*, designed to descend from a height of 80 cm.



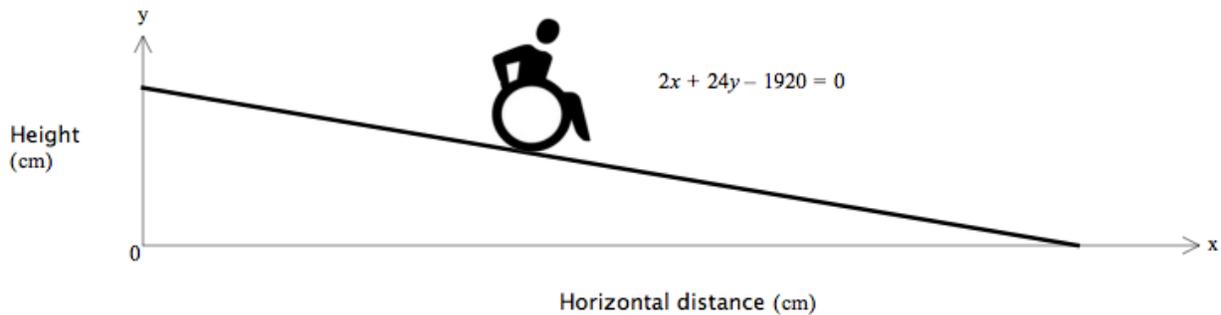
a) Use the diagram above to calculate the gradient of the ramp. (1 mark)

The gradient for a safe descending wheelchair ramp is $-\frac{1}{12}$.

b) Using your answer to part (a), comment on why wheelchair ramp *A* is not safe. (1 mark)

The equation of a second wheelchair ramp, *B*, is $2x + 24y - 1920 = 0$

Diagram not to scale

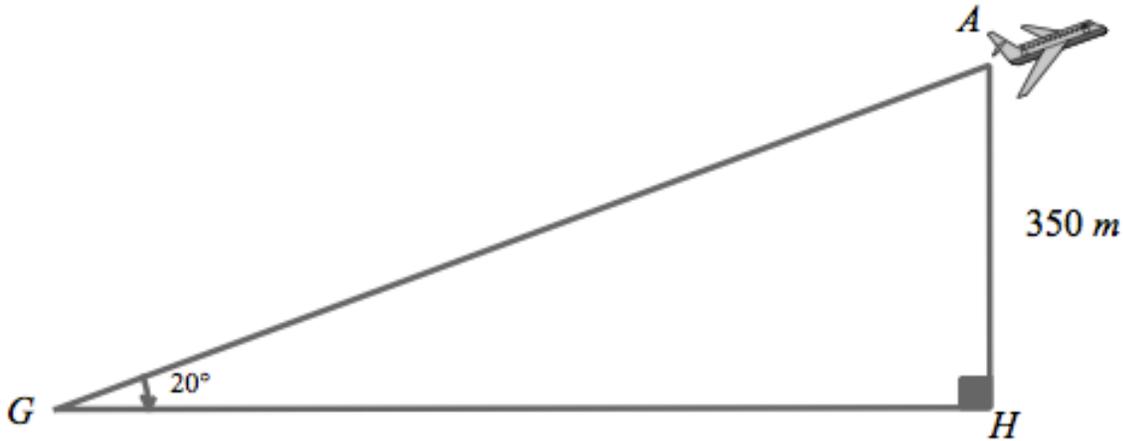


c) i. Determine whether wheelchair ramp *B* is safe or not. Justify your answer.

ii. Find the horizontal distance of wheelchair ramp *B*. (4 marks)

8. Gunther is at Berlin Tegel Airport watching the planes take off. He observes a plane that is at an angle of elevation of 20° from where he is standing at point G . The plane is at a height of 350 metres. This information is shown in the following diagram.

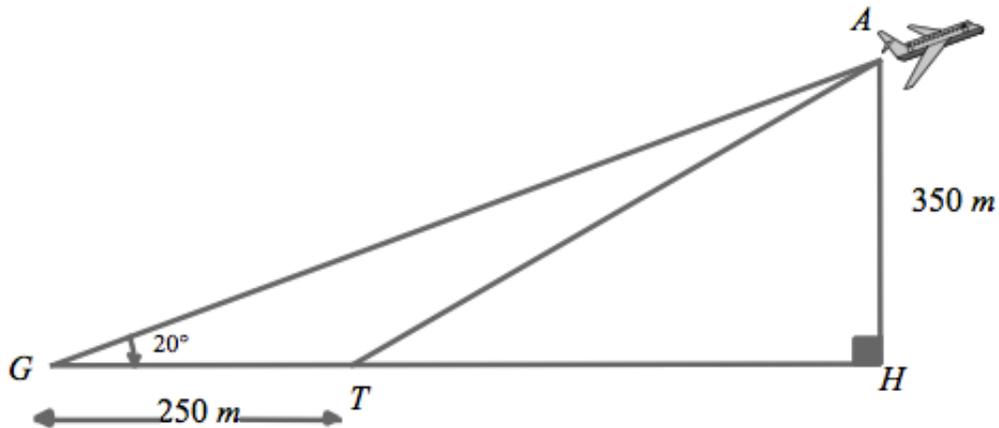
Diagram is not to scale



- a) Calculate the horizontal distance, GH , of the plane from Gunther. **Give your answer to the nearest metre.** (3 marks)

The plane took off from a point T , which is 250 metres from where Gunther is standing, as shown in the following diagram.

Diagram not to scale



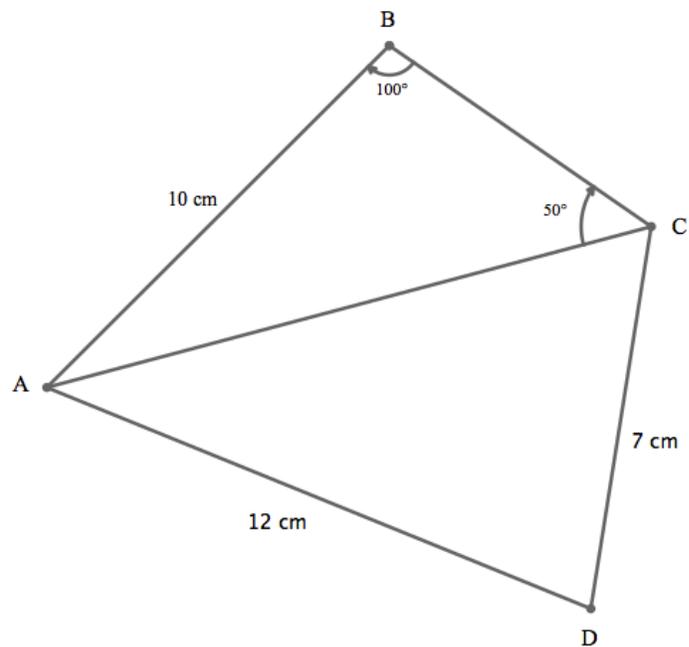
- b) Using your answer from part (a), calculate the angle ATH , the takeoff angle of the plane. (3 marks)

9. A cuboid has the following dimensions: length = 8.7 cm, width = 5.6 cm and height = 3.4 cm.

- a) Calculate the exact value of the volume of the cuboid, in cm^3 . (2 marks)
- b) Write your answer to part (a) correct to
 - i. One decimal place:
 - ii. Three significant figures. (2 marks)
- c) Write your answer to **part (b)(ii)** in the form $a \times 10^k$, where $1 \leq k < 10, k \in \mathbb{Z}$ (2 marks)

10. The quadrilateral $ABCD$ has $AB = 10 \text{ cm}$ and $CD = 7 \text{ cm}$. The size of angle ABC is 100° and the size of angle ACB is 50° .

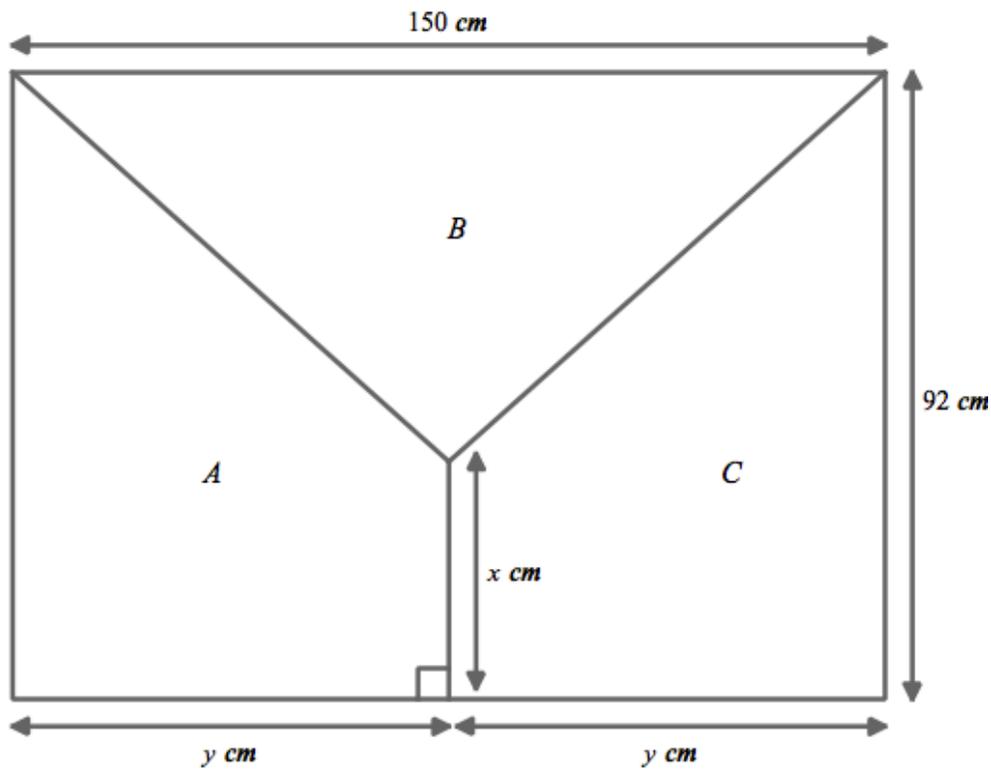
Diagram not to scale



- a) Find the length of AC in centimetres. (3 marks)
- b) Find the size of angle ADC . (3 marks)

11. The diagram below represents a rectangular flag with dimensions 150 cm by 92 cm. The flag is divided into three regions A, B and C.

diagram not to scale



a) Write down the total area of the flag. (1 mark)

b) Write down the value of y . (1 mark)

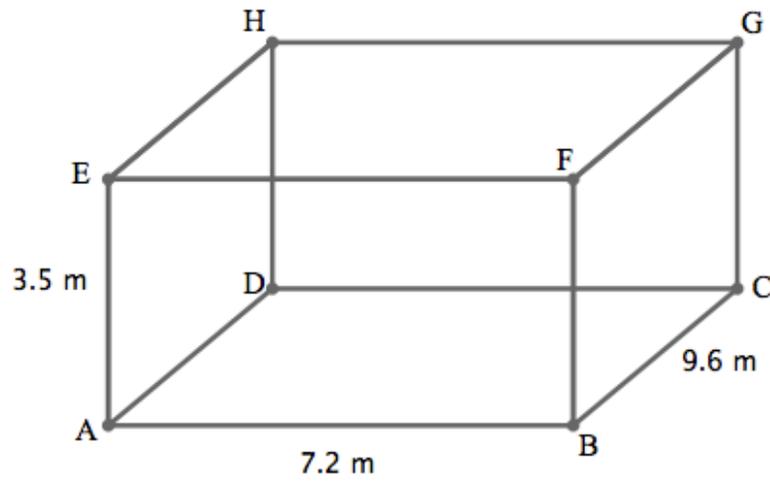
The areas of regions A, B and C are equal.

c) Write down the area of region A. (1 mark)

d) Use your answers to parts (b) and (c), find the value of x . (3 marks)

12. A room is in the shape of a cuboid. Its floor measures 7.2 m by 9.6 m and its height is 3.5 m .

diagram not to scale



a) Calculate the length of AC . (2 marks)

b) Calculate the length of AG . (2 marks)

c) Calculate the angle that AG makes with the floor. (2 marks)