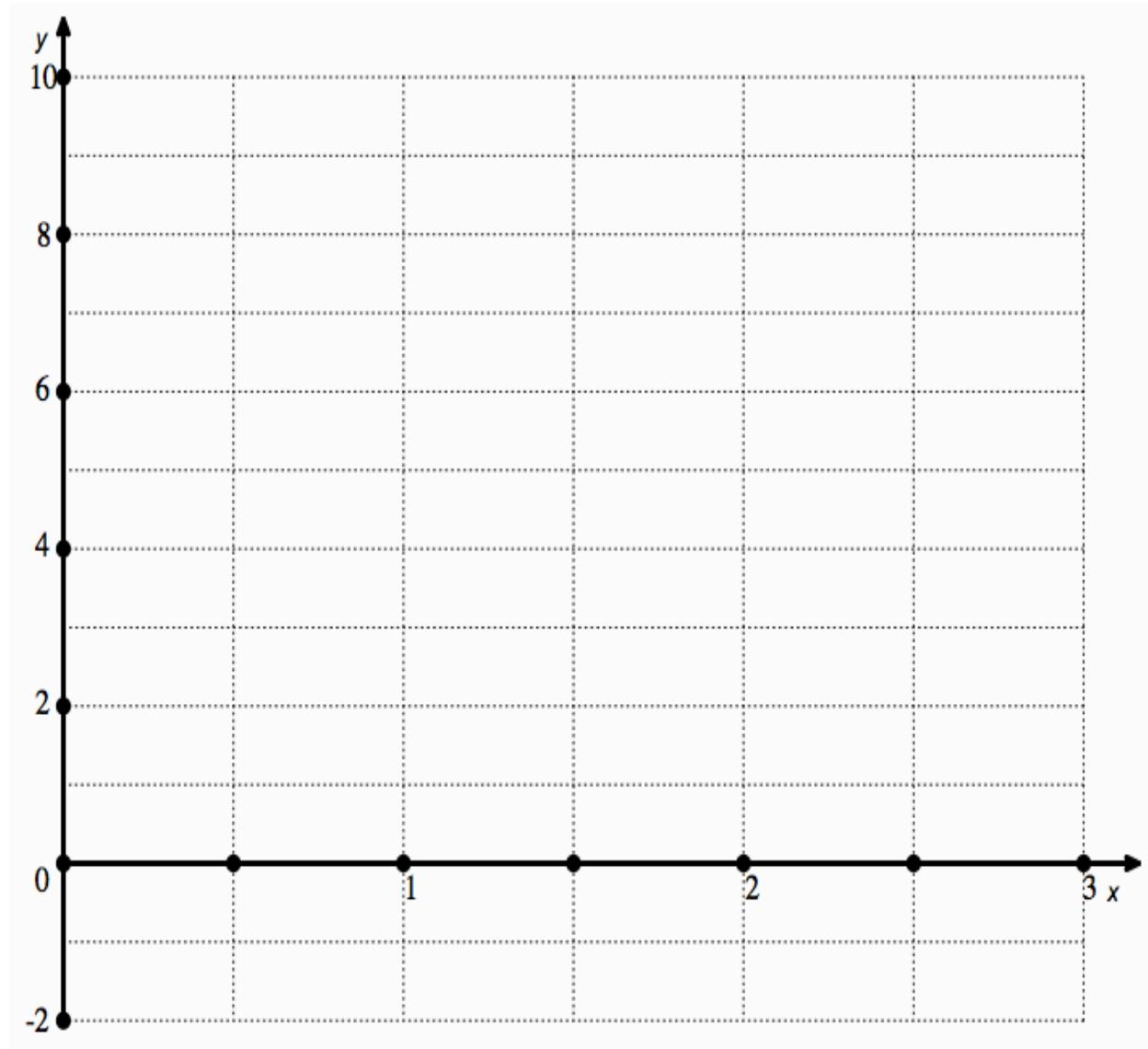


## Topic 6: Mathematical Models

Topic number	Content
6.1	Concept of a function, domain, range and graph. Function notation such as $f(x)$ , $v(t)$ , $C(n)$ Concept of a function as a mathematical model
6.2	Linear models. Linear functions and their graphs, $f(x) = mx + c$
6.3	Quadratic models. Quadratic functions and their graphs (parabolas), $f(x) = ax^2 + bx + c; a \neq 0$ Properties of a parabola: symmetry; vertex; intercepts on the $x$ -axis and $y$ -axis. Equation of the axis of symmetry, $x = -\frac{b}{2a}$
6.4	Exponential models Exponential functions and their graphs: $f(x) = ka^x + c: a \in \mathbb{Q}^+, a \neq 1, k \neq 0$ $f(x) = ka^{-x} + c: a \in \mathbb{Q}^+, a \neq 1, k \neq 0$ Concept and equation of a horizontal asymptote
6.5	Models using functions of the form $f(x) = ax^m + bx^n + \dots: m, n \in \mathbb{Z}$ Functions of this type and their graphs. The $y$ -axis as a vertical asymptote.
6.6	Drawing accurate graphs. Creating a sketch from information given. Transferring a graph from graphing calculator to paper. Reading, interpreting and making predictions using graphs. Included all the functions above and additions and subtractions.
6.7	Use of a graphing calculator to solve equations involving combinations of the functions above.

1.

- a) On the grid below sketch the graph of the function  $f(x) = 2(1.6)^x$  for the domain  $0 \leq x \leq 3$  (2 marks)



- b) Write down the coordinates of the y-intercept of the graph of  $y = f(x)$  (1 mark)
- c) On the grid draw the graph of the function  $g(x) = 5 - 2x$  for the domain  $0 \leq x \leq 3$  (2 marks)
- d) Use your graphic display calculator to solve  $f(x) = g(x)$ . (1 mark)

2. A liquid is heated so that after 20 seconds of heating its temperature,  $T$ , is  $25^{\circ}\text{C}$  and after 50 seconds of heating its temperature is  $37^{\circ}\text{C}$ .

The temperature of the liquid at time  $t$  can be modeled by  $T = ax + b$ , where  $t$  is the time in seconds after the start of heating.

Using this model one equation that can be formed is  $20a + b = 25$

- a) Using the model, write down a second equation in  $a$  and  $b$ . (2 marks)
  
- b) Using your graphic display calculator or otherwise, find the value of  $a$  and of  $b$ . (2 marks)
  
- c) Use the model to predict the temperature of the liquid 60 seconds after the start of heating. (2 marks)

3. A small manufacturing company makes and sells  $x$  machines each month. The monthly cost  $C$ , in dollars, of making  $x$  machines is given by

$$C(x) = 2600 + 0.4x^2$$

The monthly income  $I$ , in dollars, obtained by selling  $x$  machines is given by

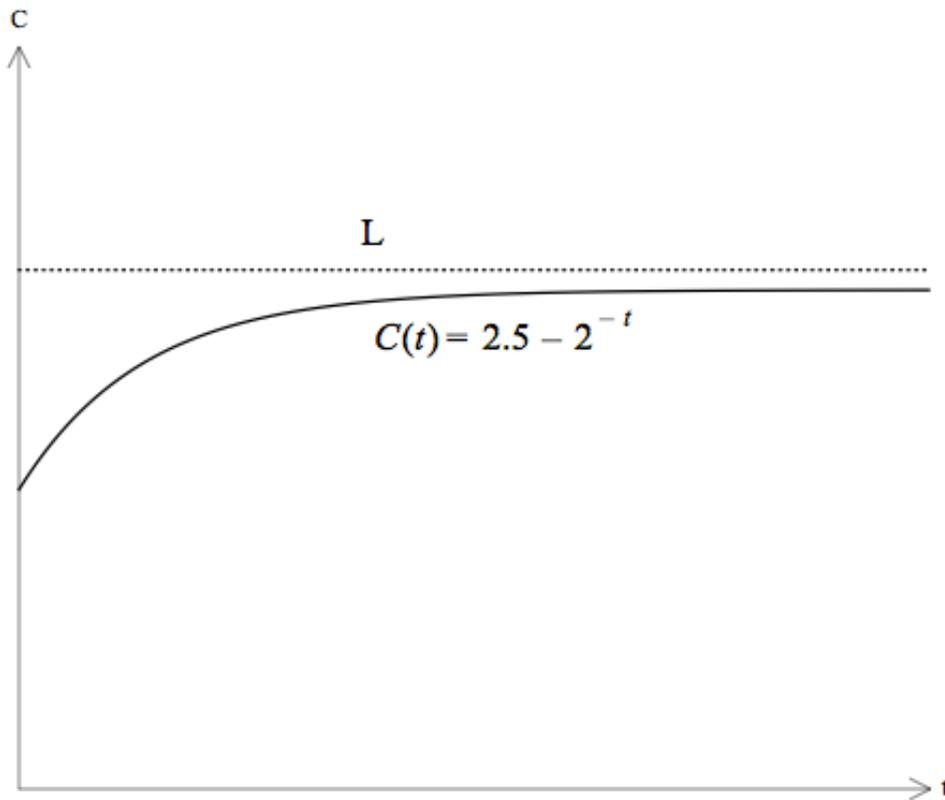
$$I(x) = 150x - 0.6x^2$$

$P(x)$  is the monthly profit obtained by selling  $x$  machines.

- a) Find  $P(x)$  (2 marks)
  
- b) Find the number of machines that should be made and sold each month to maximize  $P(x)$  (2 marks)
  
- c) Use your answer to part (b) to find the selling price of **each machine** in order to maximize  $P(x)$  (2 marks)

4. The amount of electrical charge,  $C$ , stored in a mobile phone battery is modeled by  $C(t) = 2.5 - 2^{-t}$ , where  $t$ , in hours, is the time for which the battery is being charged.

*Diagram not to scale*



a) Write down the amount of electrical charge in the battery at  $t = 0$ . (1 mark)

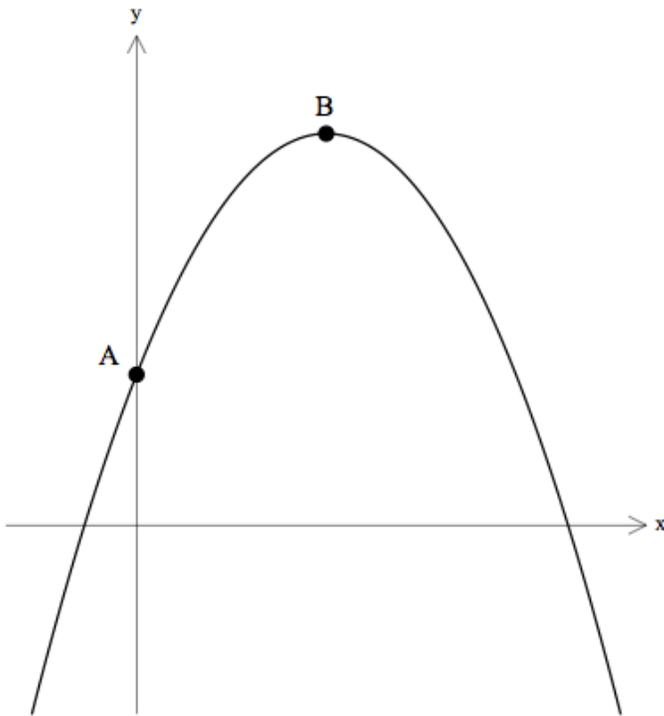
The line  $L$  is the horizontal asymptote to the graph.

b) Write down the equation of  $L$ . (2 marks)

To download a game to the mobile phone, an electrical charge of 2.4 units is needed.

c) Find the time taken to reach this charge, Give your answer correct to the nearest minute. (3 marks)

5. The graph of the quadratic function  $f(x) = ax^2 + bx + c$  intersects the y-axis at point  $A(0, 5)$  and has its vertex at point  $B(4, 13)$



- a) Write down the value of  $c$ . (1 mark)
- b) By using the coordinates of the vertex  $B$ , or otherwise, write down two equations in  $a$  and  $b$ . (3 marks)
- c) Find the value of  $a$  and of  $b$ . (2 marks)

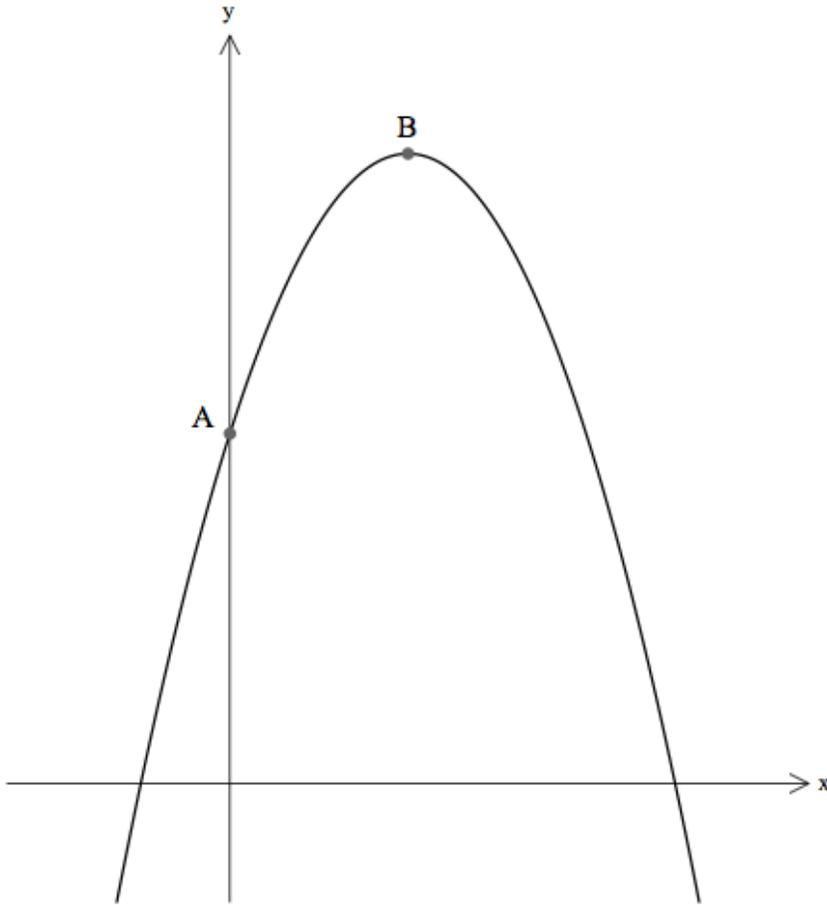
6. In a trial for a new drug, scientists found that the amount of the drug in the bloodstream decreased over time, according to the model

$$D(t) = 1.2 \times (0.87)^t, t \geq 0$$

where  $D$  is the amount of the drug in the bloodstream in mg per litre ( $mg\ l^{-1}$ ) and  $t$  is the time in hours.

- a) Write down the amount of the drug in the bloodstream at  $t = 0$ . (1 mark)
- b) Calculate the amount of the drug in the bloodstream after 3 hours. (2 mark)
- c) Use your graphic display calculator to determine the time it takes for the amount of the drug in the bloodstream to decrease to  $0.333\ mg\ l^{-1}$  (3 marks)

7. The graph of the quadratic function  $f(x) = c + bx - x^2$  intersects the  $y$ -axis at point  $A(0, 5)$  and has its vertex at point  $B(2, 9)$



a) Write down the value of  $c$ . (1 mark)

b) Find the value of  $b$ . (2 marks)

c) Find the  $x$ -intercepts of the graph of  $f$ . (2 marks)

d) Write down  $f(x)$  in the form  $f(x) = -(x - p)(x + q)$  (1 mark)

8. The straight line,  $L_1$ , has equation  $2y - 3x = 11$ . The point  $A$  has coordinates  $(6, 0)$ .

a) Give a reason why,  $L_1$  **does not** pass through  $A$ . (1 mark)

b) Find the gradient of,  $L_1$ . (2 marks)

$L_2$  is a line perpendicular to  $L_1$ . The equation of,  $L_2$  is  $y = mx + c$ .

c) Write down the value of  $m$ . (1 mark)

$L_2$  does pass through  $A$ .

d) Find the value of  $c$ . (2 marks)

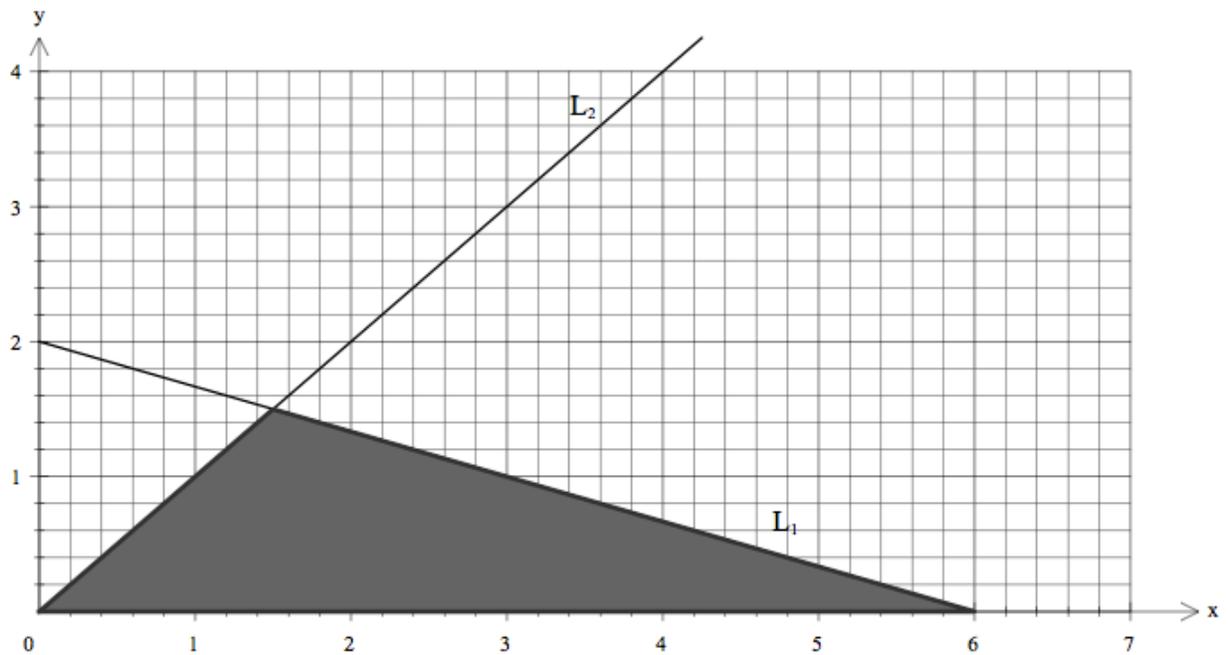
9. The number of bacteria in a colony is modeled by the function

$$N(t) = 800 \times 3^{0.5t}, t \geq 0$$

where  $N$  is the number of bacteria and  $t$  is the time in hours.

- a) Write down the number of bacteria in the colony at time  $t = 0$ . (1 mark)
- b) Calculate the number of bacteria present at 2 hours and 30 minutes, Give your answer correct to the nearest hundred bacteria. (3 marks)
- c) Calculate the time, in hours, for the number of bacteria to reach 5500. (2 marks)

10. The diagram shows the straight lines  $L_1$  and  $L_2$ . The equation of  $L_2$  is  $y = x$ .



a) Find

i. The gradient of  $L_1$ ;

ii. The equation of  $L_1$ ; (3 marks)

b) Find the area of the shaded triangle. (3 marks)