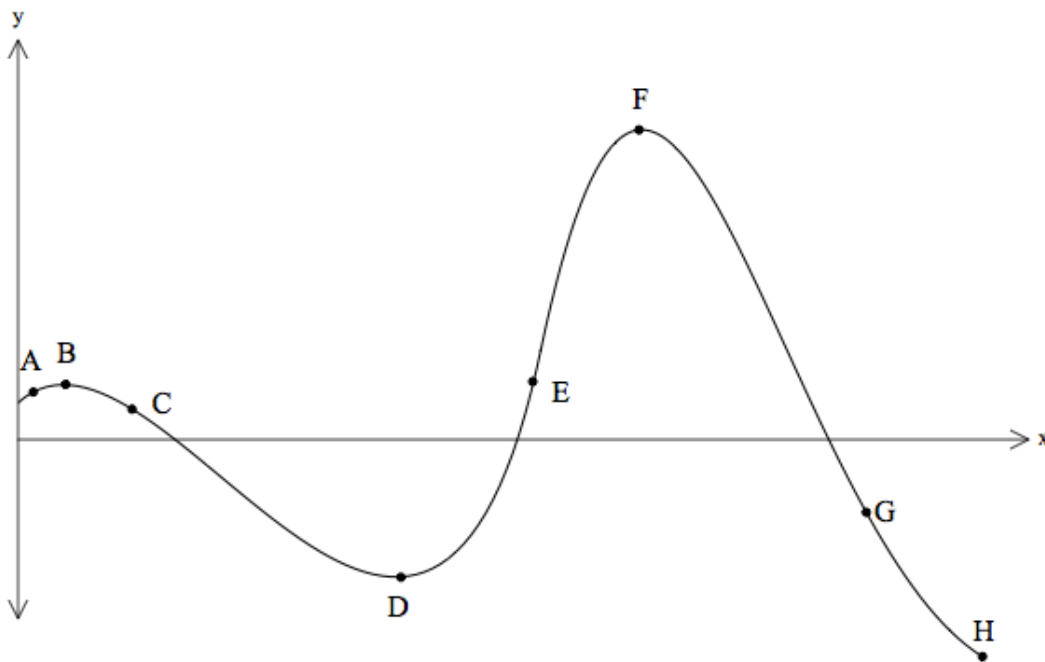


Topic 7: Introduction to differential calculus

Topic number	Contents
7.1	Concept of the derivative as a rate of change. Tangent to a curve.
7.2	The principle that $f(x) = ax^n \Rightarrow f'(x) = anx^{n-1}$ The derivative of functions of the form $f(x) = ax^n + bx^{n-1} + \dots$ where all exponents are integers.
7.3	Gradients of curves for given values of x . Values of x where $f'(x)$ is given. Equation of the tangent at a given point. Equation of the line perpendicular to the tangent at a given point(normal).
7.4	Increasing and decreasing functions. Graphical interpretation of $f'(x) > 0$, $f'(x) = 0$ and $f'(x) < 0$.
7.5	Values of x where the gradient of a curve is zero. Solution of $f'(x) = 0$ Stationary points. Local maximum and minimum points.
7.6	Optimization problems

1. Consider the graph of the function $y = f(x)$ defined below.



Write down **all** the labeled points on the curve

- a) That are local maximum points; (1 mark)
- b) Where the function attains its least value; (1 mark)
- c) Where the function attains its greatest value; (1 mark)
- d) Where the gradient of the tangent to the curve is positive; (1 mark)
- e) Where $f(x) > 0$ and $f'(x) < 0$. (2 marks)

2. Consider the curve $y = x^2$.

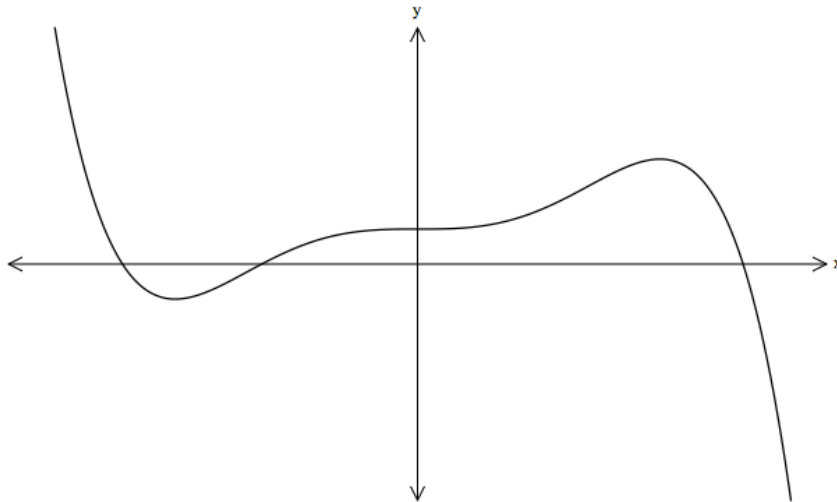
a) Write down $\frac{dy}{dx}$. (1 mark)

The point $P(3, 9)$ lies on the curve $y = x^2$.

b) Find the gradient of the tangent to the curve at P . (2 marks)

c) Find the equation of the normal to the curve at P . Give your answer in the form $y = mx + b$ (3 marks)

3. A sketch of the function $f(x) = 5x^3 - 3x^5 + 1$ is shown for $-1.5 \leq x \leq 1.5$ and $-6 \leq y \leq 6$.



a) Write down $f'(x)$. (2 marks)

b) Find the equation of the tangent to the graph of $y = f(x)$ at $(1, 3)$ (2 marks)

c) Write down the coordinates of the second point where this tangent intersects the graph of $y = f(x)$. (2 marks)

4. Let $f(x) = x^4$.

a) Write down $f'(x)$. (1 mark)

Point $P(2, 16)$ lies on the graph of f .

b) Find the gradient of the tangent to the graph of $y = f(x)$ at P . (2 marks)

c) Find the equation of the normal to the graph at P . Give your answer in the form $ax + by + d = 0$, where a , b and d are integers. (3 marks)

5. Consider the curve $y = x^3 + kx$.

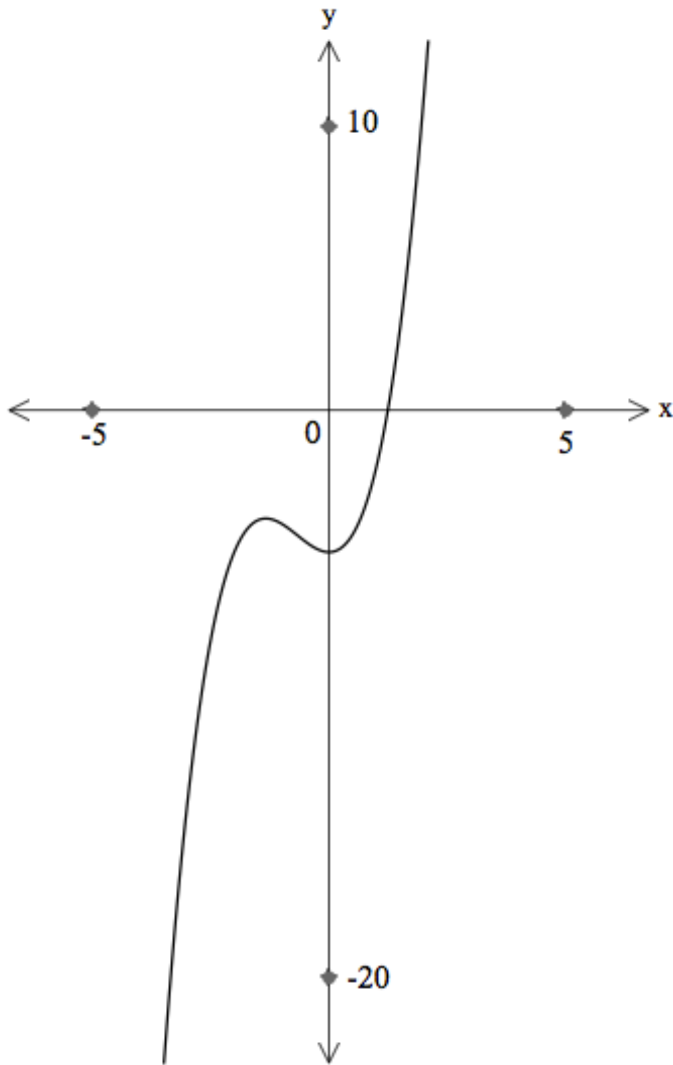
a) Write down $\frac{dy}{dx}$. (1 mark)

The curve has a local minimum at the point where $x = 2$.

b) Find the value of k . (3 marks)

c) Find the value of y at this local minimum. (2 marks)

6. Consider the graph of the function $f(x) = x^3 + 2x^2 - 5$.



- a) Label the local maximum as A on the graph. (1 mark)
- b) Label the local minimum as B on the graph. (1 mark)
- c) Write down the interval where $f'(x) < 0$. (1 mark)
- d) Draw the tangent to the curve at $x = 1$ on the graph, (1 mark)
- e) Write down the equation of the tangent at $x = 1$. (2 marks)

7. A function is given as $f(x) = 2x^3 - 5x + \frac{4}{x} + 3$, $-5 \leq x \leq 10$, $x \neq 0$

a) Write down the derivative of the function. (4 marks)

b) Use your graphic display calculator to find the coordinates of the local minimum point of $f(x)$ in the given domain. (2 marks)